I. C. 6948 September 1937

DEPARTMENT OF THE INTERIOR

UNITED STATES BUREAU OF MINES
JOHN W. FINCH, DIRECTOR

INFORMATION CIRCULAR

# AERIAL TRAMWAYS IN THE METAL-MINING INDUSTRY

## PART 1

RECEIVED
OCT 30 1937
O. S. U. LIBRARY

BY

O. H. METZGER

AFTER THIS REPORT HAS SERVED YOUR PURPOSE AND IF YOU HAVE NO FURTHER NEED FOR IT, PLEASE RETURN IT TO THE BUREAU OF MINES, USING THE OFFICIAL MAILING LABEL ON THE INSIDE OF THE BACK COVER.

I. C. 6948
September 1937

# INFORMATION CIRCULAR

## DEPARTMENT OF THE INTERIOR - BUREAU OF MINES

---

# AERIAL TRAMWAYS IN THE METAL MINING INDUSTRY - PART 1[1]

By O. H. Metzger[2]

## CONTENTS

Page

Introduction.................................... 2
Acknowledgments.................................. 2
History.......................................... 2
Types of tramways................................ 3
    Automatically operated tramways.............. 5
General considerations........................... 5
    Applications and limitations................. 5
        Maximum length........................... 6
        Factors determining capacity............. 7
    Cable anchorages, tension, and anchorage stations.......... 7
    Types of towers.............................. 8
    Rail structures.............................. 9
    Terminals and operating machinery............ 9
    Gage......................................... 9
    Construction materials....................... 9
    Equipment.................................... 10
        Carriers................................. 10
        Types of cables and couplings............ 10
        Saddles.................................. 11
        Grips.................................... 11
Design of aerial tramways........................ 12
    Determination of size of cables.............. 12
    Effect of weight and tension of traction cable.......... 16
    Calculations of cable deflections............ 18
        Corrections for variation in tension and concentrated loading.......... 19
        Comparison of methods of calculations.... 24
    Determination of height and location of towers.......... 24
        Calculation of weight on towers.......... 25
    Design of rail structures.................... 26
        High and low rollers..................... 29
    Special problems............................. 30
Power............................................ 30
Conditions affecting construction costs.......... 31
Conclusion....................................... 32

---

[1] The Bureau of Mines will welcome reprinting of this paper, provided the following footnote acknowledgment is used: "Reprinted from U. S. Bureau of Mines Information Circular 6948."

[2] Assistant mining engineer, U. S. Bureau of Mines, Mining Division, metal-mining methods section, Tuscon, Ariz.

# ILLUSTRATIONS

Figure ........ Following page

1 Double tension station........ 7
2 Double anchor station........ 7
3 Pyramid tower........ 8
4 Through tower........ 8
5 Hillside tower, through type........ 8
6 Underground loading terminal........ 9
7 Discharge terminal........ 9
8 Typical tramway profile........ 13
9 Relative position of traction cable for overhead and low rollers on a horizontal span........ 17
10 Force diagram........ 17
11 Typical rail structure location........ 27
12 Sketch showing relative position of traction cable and carrier for overhead rollers and low rollers on rail structure of short radius........ 27
13 Force diagram of figure 12 for low rollers........ 29
14 Hold-down rollers used at terminals at the lower end of steeply inclined spans........ 29

## INTRODUCTION

This is the first of two papers dealing with construction and use of aerial tramways in the metal-mining industry. It describes practices and equipment used in tramway construction and discusses the design of tramways. The second paper will describe the construction and operation of tramways and give operating costs.

## ACKNOWLEDGMENTS

The author acknowledges the cooperation of a number of cable and tramway manufacturers who supplied practical manufacturing data and other valuable information. Full credit for formulas used in the design of aerial tramways is given to Edward B. Durham[3] and F. C. Carstarphen.[4]

## HISTORY

The early history of aerial tramways is obscure, but apparently the first important installation was made in Germany in 1860. This was a tramway of the continuous monocable type and was followed later on by one of the continuous bicable type. Early systems of various types were developed in Germany and England and were introduced in America in 1887.[5]

---

[3] Durham, Edward B., Aerial Tramways and Cableways: Mining Engineers' Handbook by Peele, 2d ed., 1927, vol. 2, sec. 26, p. 1745.

[4] Carstarphen, F. C. A Simple Method of Computing Deflections of a Cable Span Carrying Multiple Loads Evenly Spaced: Colorado Sch. Mines Mag., vol. 19, no. 12, December 1929, p. 15.

[5] American Steel & Wire Co. catalog for 1935.

Without doubt, the early systems were limited in capacity and efficiency and probably were used only in places to which they were peculiarly adapted. With improvement in metallurgical, construction, and engineering practices, their capacity, efficiency, and usefulness has been greatly extended, until today aerial trams are found in many industries, competing successfully with other modes of transportation.

## TYPES OF TRAMWAYS

The two types of tramways most generally used are the double-rope continuous tramway and the jig-back or reversible tramway. The double-rope tramway consists of two stationary or track cables and an endless traction cable, with a number of carriers attached at more or less equal intervals by means of grips. The traction rope runs around sheaves at the terminals and is generally operated by a motor and controlled by suitable braking equipment. Usually, the carriers are detached at the terminals for loading and unloading and reattached for dispatching. When loads are transported downhill the motor acts as a brake or control while the tram is operating, and in many instances power is generated and put back into the line.

The jig-back tram may be of the single reversible type or the double reversible type. In the single reversible type there is one track cable on which a single carrier is operated by a traction cable attached to a hoist or sheave and operated by a motor. The motor operates in one direction to transport the load from the loading terminal to the discharge terminal and is reversed to bring the empty carrier back to the loading terminal. The double reversible tramway operates the same as the single reversible tramway, except that there are two track cables with a carrier on each. It is generally operated by a single traction cable running around a sheave and with a carrier attached to each end.

Jig-backs of 150 tons per hour capacity are in use, but generally their capacity is less than this. Due to the reversing feature of their operation, they are generally more limited in capacity than are tramways of the double-rope continuous type. They can be operated at much higher speed than other types of tramways, and this compensates in part for their limitations in other respects. In order to work successfully, they should be built with the track cables at a fairly uniform inclination, which tends to limit their length. A double reversible jig-back requires about the same amount of cable as a double-rope continuous tramway, and because of this the difference in their costs may be negligible.

Distance is a limiting factor in the capacity of tramways of this type. This is because the number of carriers is limited to two. The relation of distance to capacity of a jig-back tram is similar to the relation of depth of a shaft to the hoisting capacity of a mine hoist.

A third type of tramway used occasionally is the monocable continuous tram. This consists of a single endless cable running around sheaves and with carriers spaced at about equal distances. The cable performs the double duty of supporting and propelling the loads. In some systems the carriers are detachable, while in others they are fixed to the cable.

When they are fixed, loading and unloading must be done while they are in motion. Detachable carriers are equipped with rollers so that they can be run on a rail at the terminals. The towers are equipped with rollers designed so that the carriers will pass over them.

Tramways of this type are rather common in England and Europe, but are less common in North America, except where they are used for light duty, such as carrying fruit and other light material. Systems up to 30,000 feet in length are in successful use. Some American manufacturers have discontinued building them, whereas, others who still build them maintain that they are very satisfactory for long distances and large capacities.

A very successful tram of this type is used by the Hollinger Gold Mining Co., Ontario, Canada,[6] for hauling and distributing sand and gravel for back filling in the mines. This tram, which was built by an English manufacturer, is 3.6 miles long and the difference in elevation between the loading and discharge terminals is only 100 feet. The rope speed is 527 feet per minute, and the capacity of the carriers is 1,900 pounds, the hourly capacity of the tram being 190 tons. The cable is 1-5/16 inches in diameter and is operated at a tension of 26,000 pounds. Several cables have been worn out since the tram was installed. The last cable was still in use after having transported over 1 1/2 million tons of sand. The costs are about 6 cents per ton, including tram operation, maintenance and replacement of cable, cost of unloading the buckets, and maintenance of distributing equipment, towers, etc.

This tram was built in a country where the topography is very favorable. The terrain is comparatively level, which makes it possible to space the towers very closely and hence operate the cable at a comparatively low tension for a tram of its capacity. Furthermore, the difference in elevation between the highest and lowest point - in this case, the terminals - is not enough to cause any serious increase in tension at the upper terminal. In rough country, where difficult profiles cannot be avoided and consequently close spacing of towers is impossible, the cable tension for a tram of 190 tons per hour capacity would have to be 30,000 pounds or more at the lowest point. If the difference in elevation between the highest and lowest point were 1,500 feet, which is not unusual, the tension at the highest point would be increased by 28,000 pounds, due to the weight of the loads and the cable, making a total of 58,000 pounds. This would indicate that monocable systems should be avoided in rough, steep countries because of the impossibility of avoiding excessive tension in the running cable.

Of the three types of tramways mentioned, the double-rope continuous type is the one most commonly used in the metal-mining industry. It is also the most flexible and lends itself more readily to varying topography, long distances, and large capacities than does any other type. The remainder of this paper will be devoted to a discussion of tramways of this type. Formulas used for cable deflections are applicable to tramways of any type. In general, the structures for supporting the cables are essentially the same for all types, except that rail structures, tension stations, and anchor stations would seldom, if ever, be used on tramways of the jigback type or the monocable continuous type.

---

[6] Williams, Maurice E., Procurement, Transportation, and Distribution of Backfill at the Hollinger Mine: Canadian Min. Jour., vol. 56, no. 9, September 1935, pp. 418-421.

## Automatically operated tramways

With the automatically operated tramway, a tripping device discharges the load while it is in motion, and the carrier remains gripped to the traction cable as it passes around a sheave designed for that purpose. Loading is done from chutes that have mechanically operated gates. The carriers generally are ungripped for loading, but systems are in use whereby the loading is done from a hopper that moves on overhead rails along with the carrier until the load is delivered to the carrier. In this way, loading is done without ungripping the buckets from the traction cable. There is no particular advantage in this method of loading, and it tends to complicate the system by adding more moving parts than would otherwise be necessary. Most of the heavy-duty systems used in the coal-mining industry are unloaded automatically, but they are loaded from a stationary chute while the bucket is ungripped.

Generally, only one man is required to operate tramways of this kind. The operator is stationed in the loading terminal; he starts and stops the tram and works the levers that control the mechanically operated gates. Gripping and ungripping is done automatically and the loads are dispatched by an automatic timing device.

The carriers of an automatically operated tramway can be made to discharge their load at any point along the line. This is taken advantage of in the coal-mining industry, where they are used for mine-waste disposal. Other conditions permitting, there is no reason why a tram of this kind could not be used for transporting mill tailings from the mill and discharging them at points along the line and at the same time transport ore from the mine to the mill.

At metal-mining plants, where trams are used for transporting timber, rails and other supplies to the mine as well as for transporting ore and supplies from the mine, an automatically operated tram would have no advantage over a hand-operated system.

# GENERAL CONSIDERATIONS

## Applications and Limitations

For transportation over distances up to 25 miles in regions where the topography is extremely rough, or down precipitous slopes, aerial tramways have a great advantage over most other methods of transportation. Their operation is practically unaffected by snow or other weather conditions. Construction costs may vary greatly with topography, but operating costs are affected very little. Mountain ranges are crossed and ravines and streams are spanned without expensive grading or other preliminary work.

Aerial trams have been adopted at many mines as the only practical means of transportation. In the high altitudes of Colorado and Utah, many mines are practically inaccessible during the winter months except by means of aerial trams. At these mines the trams are used for passenger service as well as for transporting all kinds of material to and from the mines. On the other hand, due to their efficiency and dependability, they are used in a great many places where other methods of transportation are feasible.

The capacity of a tramway may range from 25 to 300 tons per hour, but in the metal-mining industry their use has been limited largely to plants of less than 1,000 tons daily capacity. However, aerial trams could be used at metal-mining plants having capacities greatly in excess of this. They are used in the coal-mining industry for transporting coal and mine waste at a cost as low as 1 cent per ton mile. The construction industry utilizes them for transporting sand and gravel. In the past 10 years they have been used on many large construction projects in every part of the country as the most economical and dependable means of transportation regardless of the temporary nature of the work.

Because of their great dependability, it has been possible to operate tramways continuously and to transport as much as 6,000 tons of material every 24 hours. Tramways having greater capacities than this are in use now, and still greater capacities are planned.

The feasibility of underground loading terminals has been proved and underground construction practices have been sufficiently worked out to permit the operation of carriers and traction cable in adits or tunnels of any length. Such arrangements make possible the transfer of ore from ore passes and underground bins directly to tramway carriers, thus eliminating underground motor haulage and extra handling of ore at the portal of the haulage tunnel.

The tramway of the San Francisco Del Oro mines near Parral, Mexico, is an outstanding example of the application of aerial tramways to underground haulage problems. This tramway, described in the Engineering and Mining Journal of August 1936, runs underground in an adit for 3,000 feet.

## Maximum Length

A tramway 54 miles long is in operation in Columbia, South America, and projects are under way for still longer ones. Long tramways are built in sections, each operated by an independent traction cable passing around sheaves at the ends of each section. The carriers are automatically detached from the traction cable of one section and automatically attached to the traction cable of the next. The length of a section depends on the difference in elevation of the highest and lowest point and also on the friction necessary to be overcome by the traction cable. The track cables between sections are either continuous or connected by means of rail structures, and the carriers pass from one section to the other without interruption.

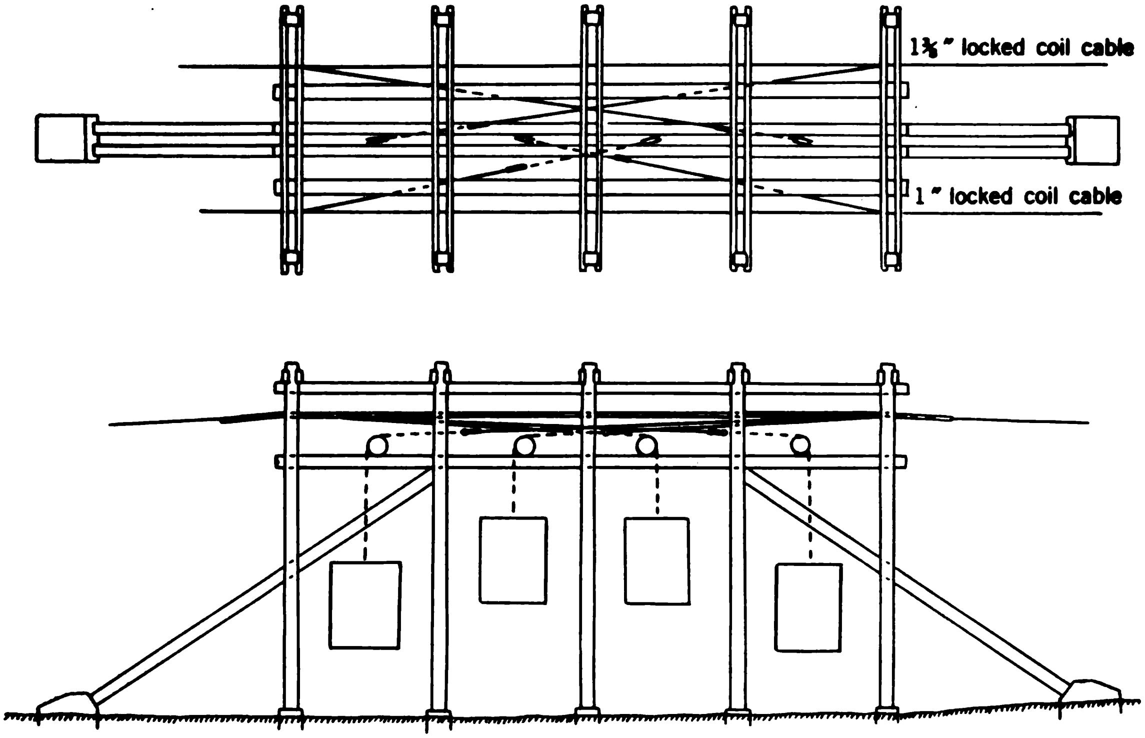


Figure 1.— Double-tension station.

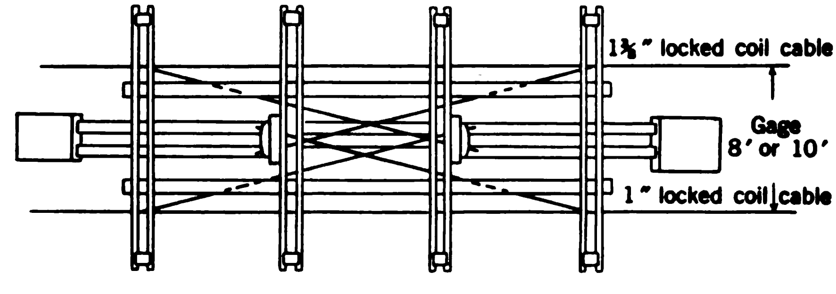


Figure 2.— Double-anchor station.

Angle stations sometimes are found to be of great advantage in countries having extremely rough topography. Instead of a straight line between terminals, an angle station permits one or more horizontal angles in the alignment, and hence makes possible a choice of more suitable topography than would otherwise be possible. An angle station may be the ends of two sections, each operated by an independent traction cable, or the traction cable may pass through the angle station to the other part of the tram, making a continuous system. In the former case the carriers would have to be detached and attached, as in the case of a long tramway built in sections. In the latter, the carriers remain gripped as the traction cable passes around sheaves designed for that purpose.

## Factors Determining Capacity

The three factors that determine the capacity of a tram are the capacity of the individual carriers, the spacing of the carriers, and the speed of the traction cable. Carriers having a capacity of 1,500 pounds or more cause excessive wear on the track cables, due to severe bending stresses caused by the rolling load, which is usually carried on two wheels. In the heavy-duty trams, these bending stresses have been reduced to a great extent by using carriers with four wheels, thus distributing the load over four points on the track cable instead of two. Extremely close spacing and high rope speeds are objectionable because of the difficulties involved in handling the carriers at the terminals. With trams that operate automatically the rope speed should be less than the highest speed of hand-operated trams, but the spacing may be such that the carriers enter and leave ther terminals at the rate of four or five a minute. With hand-operated trams, the rope speed should not exceed 600 feet a minute and the rate at which the carriers enter and leave the terminals should not exceed three a minute. Twenty tons an hour is about the low economic limit for tramways of the double-rope continuous type.

## Cable Anchorages, Tension, and Anchorage Stations

The general practice in tramway construction is to anchor one end of a track cable in concrete or to a suitably constructed anchor station, and to secure the desired tension by suspending a weight at a tension station or at one of the terminals. With tramways up to 5,000 feet long cables can be anchored at one terminal and weights suspended at the other. In the case of longer tramways, tension stations or anchor stations or both must be used. Figure 1 shows a double-tension station and figure 2 a double-anchor station. The number and location of these stations depend on the length of the tram and the topography of the region in which it is built.

Cables are supported by towers at intermediate stations along the line wherever necessary. For tramways of moderate capacity, these towers generally have but one point of support. The cables lie loosely in saddles and can slide forward and back as the load varies.

Some tramway engineers anchor both ends of the track cables. This practice undoubtedly has its advantages, especially on lines where great variations in sag may be objectionable. In the case of weighted spans the tension remains constant but the sag varies with the load. With anchored spans the sag has much less variation, but the tension increases in nearly direct proportion with the load. Because of this, anchored spans require a heavier track cable to take care of variations in load than is required on a weighted span for the same purpose.

Where towers are required on concave profiles, as in broad valleys, the cables are likely to lift out of the saddles of the lower towers when that part of the line is unloaded. If the cables are anchored, this is less likely to happen than if they are weighted. On such profiles excessive variation in sag due to variation in load on weighted spans can be eliminated by blocking up under the weight boxes to within about a foot of their position when the line is normally loaded. Then, if the line is unloaded, the weights rest upon the blocking and only a slight variation in sag takes place. This practice provides, to a great extent, the favorable characteristics of an anchored span and has the advantage that the tension does not become excessive if the line is overloaded.

## Types of Towers

The pyramid tower (fig. 3) and the through tower (fig. 4) are the two types most generally used. The pyramid tower is economical of material and very satisfactory for steel construction. It is not so satisfactory for timber construction, as there is not sufficient area at the top for properly fastening the saddle beam.

The through tower is very sturdy and is well adapted to timber construction but requires almost twice as much material per foot of height as the pyramid tower. For heavy-duty trams it is built with nine posts, but for lighter lines the two outside center posts can be left out.

The hillside tower (fig. 5) is an adaptation of either the pyramid or the through tower for use on steep hillsides or where the inclination of the track cables is very steep. The posts on the downhill side are perpendicular, while the posts on the uphill side are on a line with or in a line a little outside of one normal to the cables.

On profiles that are concave, the towers will be spaced far apart, while on profiles that are convex, such as on mountain ridges, the towers must be spaced closely.

The difference in the slope of the cable on opposite sides of a tower at the point of contact with the saddle is known as the deflection angle at that tower. This angle should not be greater than 6° to 8°. If the deflection is greater than 8°, the wear on the cables becomes excessive and either a rail structure or extra towers should be used.

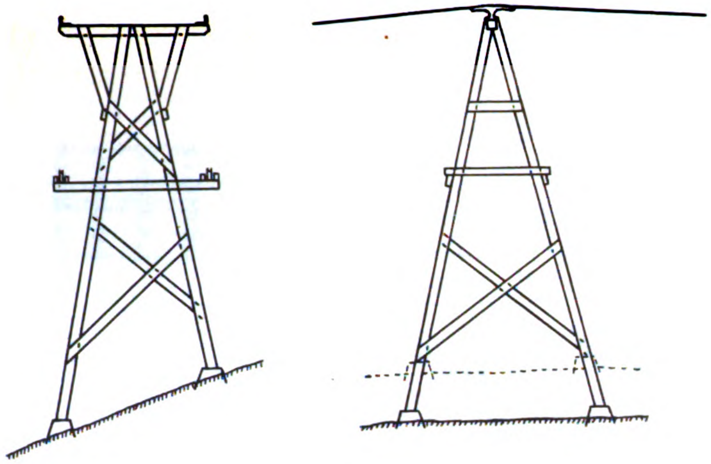


Figure 3.— Pyramid tower.

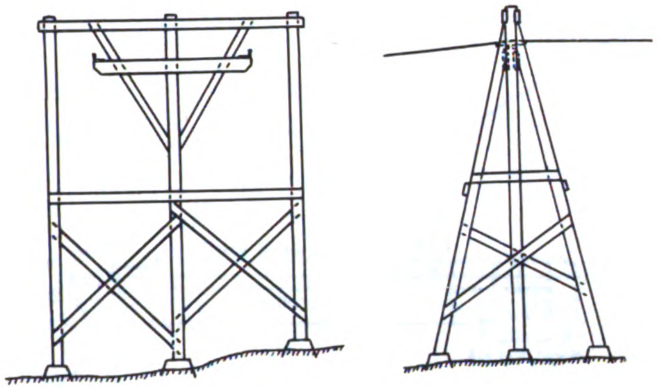


Figure 4.— Through tower.

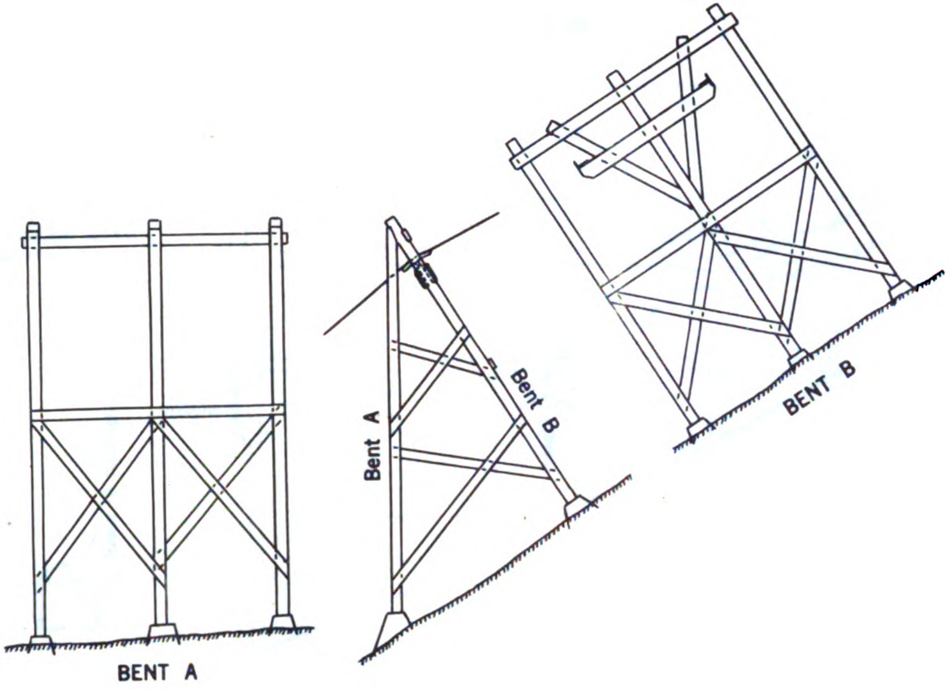


Figure 5.— Hillside tower, through type.

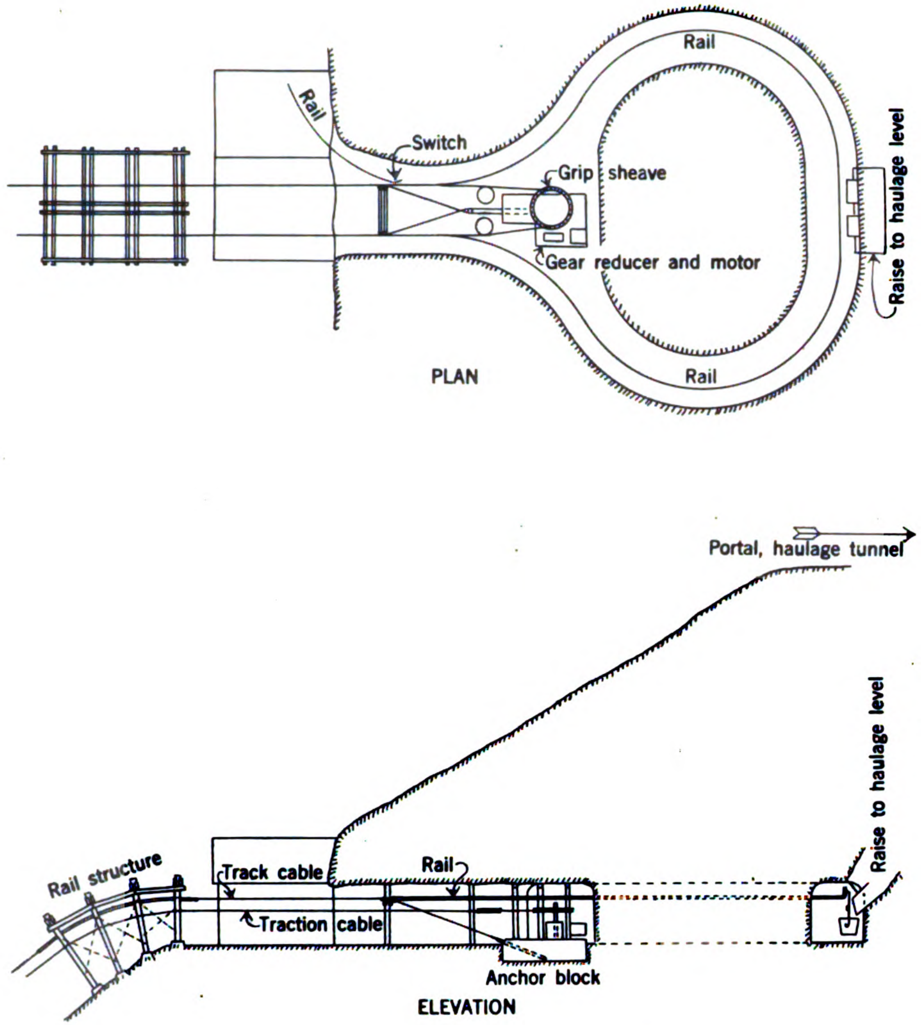


Figure 6.– Underground loading terminal.

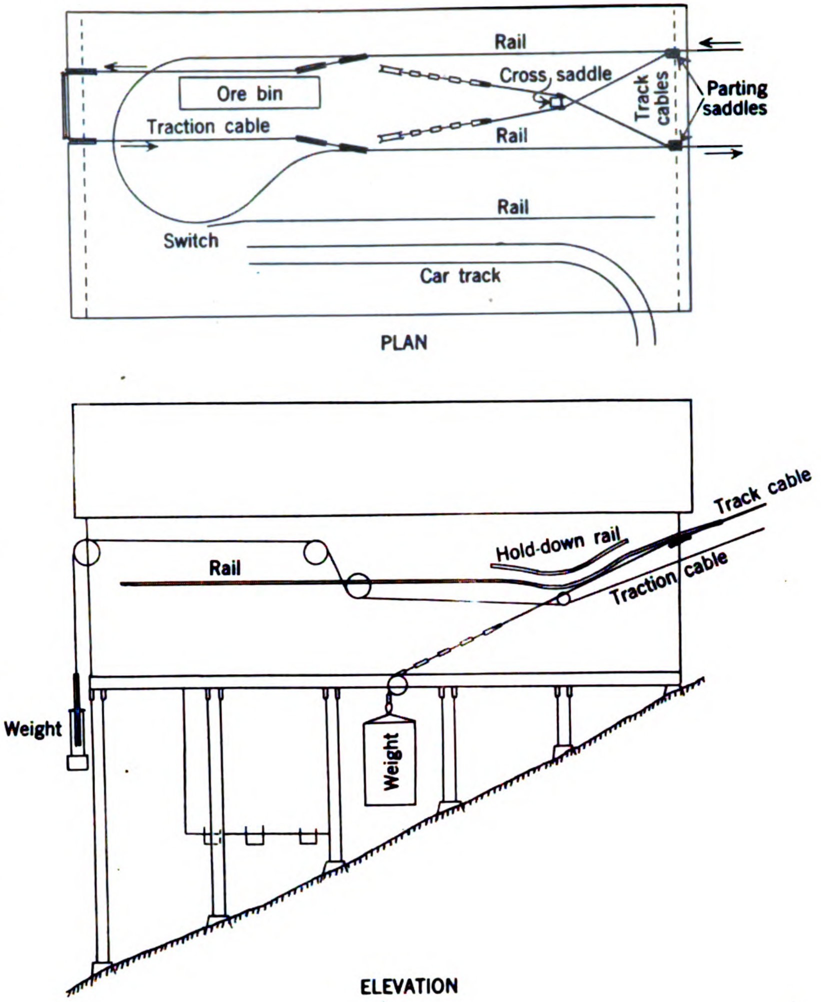


Figure 7.– Discharge terminal.

## Rail structures

Rail structures are from 15 to as much as 100 feet long. The cables are supported at points from 5 to 10 feet apart, constructed on the arc of a circular curve. A curved rail is fastened just above the cable for the carriers to move on. This saves excessive wear on the cables and provides a smooth circular track for the load to move on instead of a series of short chords. Rail structures can often be used to advantage as anchor and tension stations.

## Terminals and Operating Machinery

Terminals (figs. 6 and 7) must be provided with bins for loading and for discharging material. Where mine supplies are to be transported, sufficient floor space and switches must be provided. The carriers run on rails at the terminals, and by means of a simple throw switch they can be run on side tracks for loading heavy materials such as timber and rails. The brakes and the machinery that operates the traction cable are generally located at the loading terminal. The machinery consists principally of a sheave, around which the traction cable runs, geared or belted to a motor. The motor acts either as a drive or a control, or both.

If the difference in elevation of terminals is not great, a smooth sheave is satisfactory. Generally, two of these are mounted on the same shaft. The traction cable passes around one, then around an idler sheave, and then around the other. If there is much difference in elevation between the terminals, a grip sheave is necessary. This consists of a sheave with jaws that grip the traction rope in such manner that it will not slip if it is kept at the proper tension.

## Gage

On most modern tramways the distance between cables, or the gage, is either 8 or 10 feet. An 8-foot gage is sufficient except when the spans are extremely long, in which case a 10-foot gage would be advisable, as it would eliminate, to a great extent, the possibility of the cables and carriers swinging together during severe storms.

## Construction materials

Steel and timber both are satisfactory for tramway structures. Steel lends itself more naturally to complicated designs. It also eliminates the fire hazard, and has numerous other advantages. Present tendencies are to use steel almost entirely, except in isolated localities. The first cost is from 40 to 60 percent greater than for timber, but after a structure is once built it requires practically no attention.

Timber has certain advantages over steel, especially in places where it is impossible to get prompt deliveries of fabricated steel parts. Suitable timber generally can be obtained locally and when such material is used construction of a tram can start as soon as any part is designed.

With steel, construction must be delayed until the parts are fabricated and delivered, which is often 3 to 6 months. Such delays are serious in regions where the climate is severe and where construction work can be carried on during only 5 or 6 months of the year. Timber structures destroyed by fire or by snow slides usually can be rebuilt in from 10 to 30 days, whereas it might require months to replace a steel structure.

Timber structures require some attention after they have been completed. For the first year they should be inspected and the bolts tightened every 30 days. After the first year, this procedure is not necessary more than once every 6 months.

Concrete is preferable for foundation piers, although good masonry also is satisfactory. Concrete has an advantage in that anchor bolts are more easily placed; and usually the cost is no more than for good stone masonry. Timber sills are very unsatisfactory because they may decay and settle unevenly, throwing the towers out of plumb and alignment. With tall towers this might be very serious.

## Equipment

### Carriers

Each carrier has three parts: The carriage or trolley, the hanger, and the bucket or rack for holding the material to be carried. The buckets used in metal mining generally are pivoted on the hangers and are discharged by dumping. In some coal-mining installations the buckets discharge from the bottom, which is latched and unlatched automatically at the point of discharge. For material such as timber, pipe, and rails, special carriers must be used. These are very often ordinary carriers with the hangers bent into the form of a rack or provided with chains for fastening the material being handled. On the smaller trams two carriers are generally used for a load of timber or rails. On larger trams, where four-wheel carriers are used, only one carrier is used for a load unless the material is extremely long.

### Types of Cables and Couplings

Several kinds of cable have been developed by American manufacturers for use as track cables. The two most common are locked coil cable and smooth coil cable, both of which are satisfactory. The outside wires of locked coil cable have a cross section of the general form of the figure 8 and are interlocked in such a way that if one wire breaks it cannot work loose. The exterior surface is smooth and forms an excellent track for the carriers to roll on. It gives longer service than any other type of cable and does not cause excessive wear of saddles and carriages. Smooth coil cable is made up entirely of round wires. It has slightly greater tensile strength than lock coil cable but does not resist wear as well. Its cost is considerably less than that of lock coil, and for lines that are temporary, such as lines for construction jobs, it is just as satisfactory.

Track cables are generally made from a good grade of crucible cast steel. This is not quite as high in tensile strength as plow steel, from which hoisting cables usually are made, but it resists wear better and the wires do not break so soon. Because of their stiffness, plow-steel cables have a tendency to break down near the saddles where the bending stresses due to the rolling load are the greatest and because of repeated bending. Stranded cables should be avoided on account of the excessive wear they cause in the saddles and carriages.

Locked coil cables and smooth coil cables are made in lengths of 800 to 1,500 feet. For sections of tramway longer than this the cables are joined by couplings. These couplings consist of two round steel sockets from 8 to 12 inches long and as small in diameter as is consistent with strength. They are fastened together by a steel plug with right and left hand threads. One end of the bore of each socket is tapered so that the ends of the cable can be attached by means of wedges or molten zinc. Zinc is considered more satisfactory than wedges if the job is done under favorable conditions. If the job is done in the field under unfavorable weather conditions wedges are advisable. The couplings used for attaching the cables at tension stations, anchor stations, and terminals are similar in principal to the line couplings. They consist of only one socket for fastening the cable and are fitted with a swivel, link, or some other device for attaching to anchors or weights.

The most satisfactory traction cables are made from crucible cast steel with Lang-lay construction. The pliability of crucible cast steel and the long sections of wires exposed by Lang-lay construction make this a cable very well suited for gripping and for resisting wear, due to gripping action.

## Saddles

In the early days of tramway construction, short saddles of the stationary type were the only kind known. Although this type gave satisfactory service, the rocking type of saddle is used today almost entirely on towers having only one point of support for the cables. Some manufacturers make saddles of the rocking type 4 feet long. A saddle of this kind does much to overcome excessive bending stresses in the track cable, especially at towers where the angle of deflection is large. On rail structures, a combination stationary saddle and bracket is used. The bracket is for bolting the rail in place just above the point where the cable passes over the saddle. The carrier moves from the cable to the rail on a bonnet, one end of which is bolted to the rail and the other end of which rests over the cable. At the tension stations, anchor stations, and terminals, cross saddles and parting saddles (fig. 7) are used for deflecting the cables to points of anchorage or tension.

## Grips

The best known grips for attaching the carriers to the traction cable are of three types. In the Otto type the gripping action is obtained by means of a lever and a screw. The screw is provided with threads having a steep pitch for about two-thirds of its length, the end having threads with a very flat pitch. The grip is adjusted by means of adjusting screws, so that the threads with the steep pitch will act until the gripping jaws are in firm contact with the cable, and from there the action is taken up and the final gripping is done by the threads with flat pitch.

The Weber grip works by a lever and toggles. The Wico grip is also worked by lever and toggles, but the gripping pressure is by springs. Both of these grips can be adjusted to the size of the cable by means of screws, but the Wico grip, because of the springs, adjusts itself to places on the cable that may be abnormally large due to splicing.

The Otto grip and the Weber grip can be operated by hand for attaching, but are usually detached automatically. The Wico grip must be attached and detached automatically.

Some grips are made to open from the top, some from the bottom, and some from the side. The rollers for carrying the traction cable at the terminals must be arranged in a slightly different manner for each type of opening. Local conditions vary so greatly that the arrangement of rollers for nearly every terminal presents a different problem. In general, the rollers must be arranged on the detaching side so that the traction cable will pull away from the grip as soon as the carrier is detached. On the attaching side they must be arranged so that the grip will come in the proper position for gripping to the traction cable as the carrier is moved toward the attacher. Sometimes a dip or depression in the rail on which the carriage runs is necessary at the attacher or just ahead of the attacher.

## DESIGN OF AERIAL TRAMWAYS

Before a tramway of any kind can be designed, a careful survey must be made of the proposed route. The center line must be plotted accurately, both in elevation and horizontal plan. The locations and height of the structures are determined from the profile. A careful topographic survey is then made at each structure location, and the contours are plotted at 1- or 2-foot intervals.

In figuring the height and locations of the structures, it is first necessary to make a thorough inspection of the plotted profile. By such inspection it is possible to locate certain obvious points for structure locations and from these proceed with calculations to make the rest of the locations. After the heights and locations of the structures are determined, they are designed to fit the topography of their respective locations.

### Determination of Size of Cables

The size of the track and traction cables necessary for a tram of given capacity will be determined by the following factors:

1. The capacity of the tram per hour.
2. The speed of the traction rope.
3. The capacity and weight of the individual carriers.
4. The nature of the profile for which the tram is to be built.
5. The allowable sag or maximum deflection.

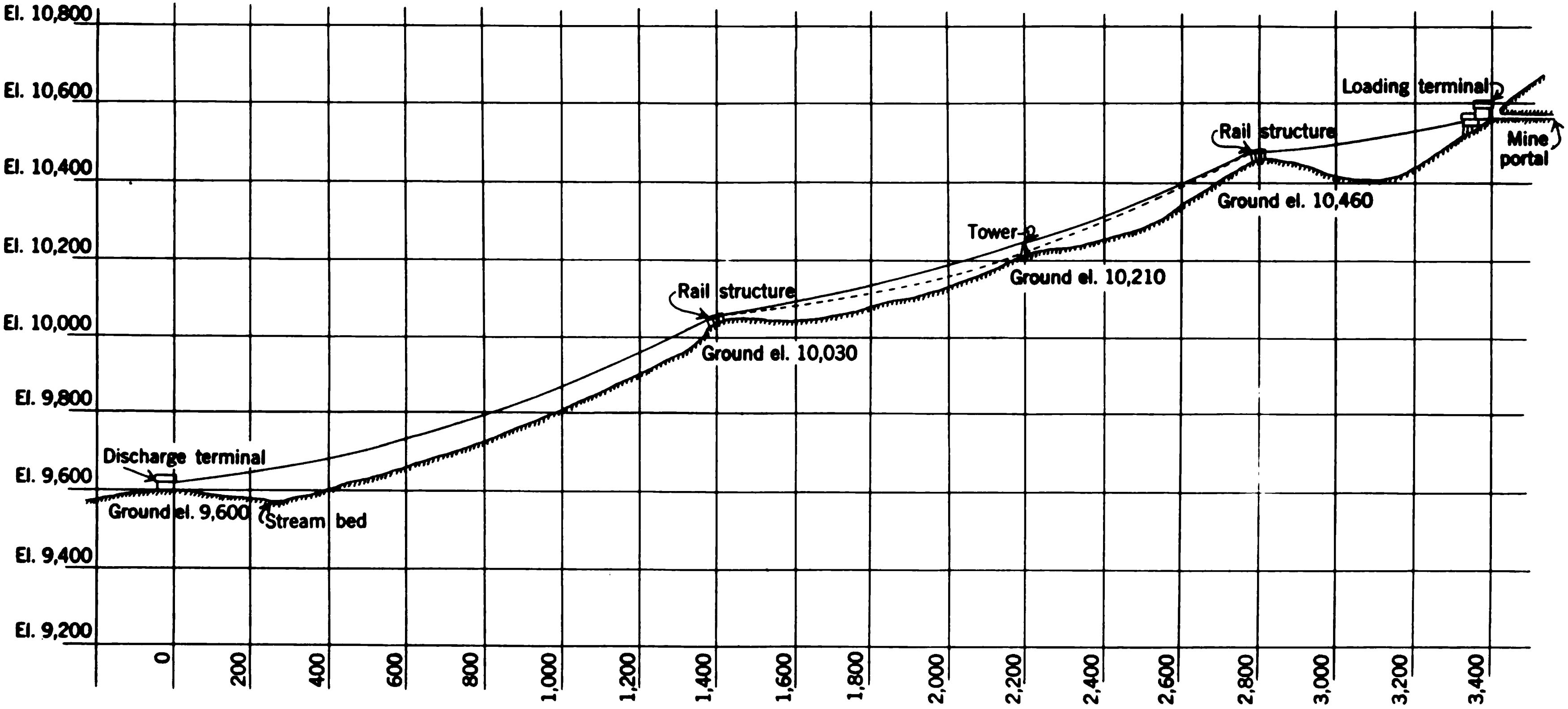


Figure 8.—Typical tramway profile. Towers located to provide a minimum clearance of 20 feet for the track cable.

Any tram permits of a certain choice in rope speed and capacity of individual carriers. Since the loading on the line varies inversely as the rope speed, the size of the track cable required will be a minimum when the rope speed is a maximum. However, high rope speeds should be avoided whenever possible because of the difficulty of handling the carriers at the terminals. In tramway construction the tendency is to compromise on speeds from 450 to 500 feet per minute, or about 8 feet per second, although higher speeds are possible.

For a tram with a capacity of 90 tons per hour, a rope speed of 8 feet per second, or 480 feet per minute, and carriers of 1,200 pounds capacity, the capacity per minute, the time interval, and the spacing are as follows:

Capacity per minute $= \frac{90 \times 2,000}{60} = 3,000$ pounds or 1-1/2 tons.

Time interval of carriers $= \frac{1,200 \times 60}{3,000} = 24$ seconds.

Spacing $= \frac{480 \times 24}{60} = 192$ feet.

If the weight of the carriers is 570 pounds and the loading is assumed to be distributed uniformly, the loading per foot due to the loaded carriers is $\frac{1,200 + 570}{192} = 9.22$ pounds. On spans where there are three or more loads, the deflection or sag is nearly the same as if the loading were distributed uniformly. For most practical purposes, it is satisfactory to assume the latter. The size of the track cable necessary to support this load will be determined by the allowable sag or maximum deflection. The greater the sag the smaller will be the size of the cable required, as the tension varies inversely as the deflection or sag for any given load on a span. The allowable sag depends upon so many practical considerations that there is no definite rule by which it can be determined. For the purpose of showing calculations it may be taken as 1/19 of the longest span.

Ordinarily, it is not known accurately which is the longest span of a tram until all structure locations are known, and this, in turn, depends on calculating cable deflections for the whole line. However, the length of the longest span generally can be determined with sufficient accuracy for preliminary calculations.

From an inspection of the profile (fig. 8), it is apparent that for the conditions shown, the supports or towers must be placed at horizontal distances of 1,400 and 2,800 feet. After the cable deflections for the spans, 0 to 1,400 and 1,400 to 2,800 are calculated and plotted, it may develop that intermediate supports are necessary, but it is evident that the longest span will be about 1,400 feet.

Considering the span from horizontal coordinate 0 to coordinate 1,400:

1/19 of 1,400 = 75 feet (approximately), which is taken to be the maximum allowable deflection for this span.

In the formula $H = \dfrac{wmn}{2t \cos \alpha}$ (1) [7]

H = 75 feet = allowable deflection at center of the span

w = total weight per foot of load and cable = 9.22 plus the weight of the cable per foot

m = distance of the point from left support = 700

n = distance of the point from right support = 700

t = working tension of cable

$\alpha$ = the vertical angle of the inclined chord

$$= \tan^{-1} \frac{\text{diff. elevation}}{1,400}$$

$$= \tan^{-1} \frac{10,030 - 9,600}{1,400} = 17° \; 04'$$

The weight of locked coil cable per square inch of cross-section area per foot is 3.1 pounds (determined from data furnished by manufacturers).

The tensile strength of locked coil cable per square inch of cross-section area is 112,000 pounds and the working tension is 32,000 pounds (safety factor 3.5) (determined from data furnished by manufacturers).

Let A = the cross-section area of the cable required, square inches

Then 3.1A = weight per foot

and 32,000A = working tension.

Substituting in formula (1)

$$75 = \frac{(9.22 + 3.1A)\,(700 \times 700)}{2 \times 32,000A \cos 17° \; 04'}$$

$$= \frac{4,517,800 + 1,519,000A}{61,184A}$$

A = 1.472 square inches

If d = diameter of cable

$\frac{\pi d^2}{4}$ = A = 1.472 square inches and d = 1.37 inches or 1-3/8 inches for the loaded side.

---

[7] Peele, Robert, Formula(18): Mining Engineers' Handbook, 2d ed., 1927, p.1749.

For the empty side, the distributed weight of the empty carriers is $\frac{570}{192} = 2.97$ pounds per foot.

Substituting in formula (1)

$$75 = \frac{(2.97 + 3.1A)\ (700 \times 700)}{2 \times 32{,}000A \cos 17^\circ\ 04'}$$

$$= \frac{1{,}455{,}300 + 1{,}519{,}000A}{61{,}184A}$$

$$A = 0.475 \text{ square inch} = \frac{\pi d^2}{4}$$

$d = 0.777$ inch = the calculated diameter of the cable required on the empty side. Since locked coil cables are made in sizes to the nearest 1/8 inch and 0.777 inch is the minimum diameter for safe working conditions, the nearest size above must be used, which is 7/8 inch. If heavy supplies, such as timber and rails, are to be hauled on the empty side, it would be advisable to use a cable of 1-inch diameter rather than one of 7/8-inch diameter.

The tension in the traction cable is a minimum at the lowest point of the profile on the empty side and a maximum at the highest point on the loaded side. Tension is applied at one of the terminals, usually the lower, to keep the cable taut. The maximum tension is equal to the applied tension at the lower terminal, plus the tension component of the weight of the cable and loaded carriers, plus or minus the tension required to overcome frictional resistance on the line. Tension required to overcome frictional resistance is generally less than 1 percent of the total and can be neglected. It is plus where the cable moves up and minus where it moves down.

Let $T_m$ = maximum tension, that is the tension at the highest point on the loaded side

$T_o$ = tension applied at lower terminal

$V_l$ = tension component of loads

$V_c$ = tension component of cable

$T_f$ = tension required to overcome friction

Then $T_m = T_o + V_l + V_c \pm T_f$. (2)

In figure 3 the discharge terminal is at the lowest point and the loading terminal at the highest point. The difference in elevation between cable supports is 10,560 - 9,620 = 940 feet. The tension component of the loa carriers = the distributed weight per foot of the loaded carriers multiplied by the difference in elevation[8] = 9.22 x 940 = 8,667 pounds. The tension component of the traction cable = the weight of the cable per foot multiplied by the difference in elevation = 940 w, in which w = weight of cable per foot. If frictional resistance is neglected and the tension applied at the lower terminal is 1,000 pounds, formula (2) becomes

$$Tm = 1{,}000 + 8{,}667 + 940w.$$

Extra strong 6 by 7 Lang-lay cable, which is used on tramways, weighs 1.90 pounds per foot per square inch and has a working tensile strength of 23,600 pounds per square inch. (Determined from data furnished by manufacturers).

Let A = the cross-section area of the cable required.

Then $Tm = 23{,}600A$

and $w = 1.90A$

Substituting in the formula

$$23{,}600A = 1{,}000 + 8{,}667 + 940 \times 1.90A,$$

$A = 0.443$ square inch.

If d = diameter of the cable,

$$\frac{\pi d^2}{4} = 0.443,$$

$d = 0.75$ or $\frac{3}{4}$ inch.

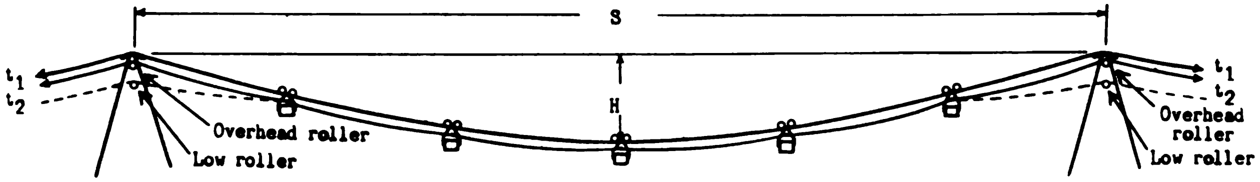
## Effect of weight and tension of traction cable

On spans at the lower end of a system where the tension in the traction cable is a minimum, most of the weight of carriers is supported by the track cable. On some of the spans higher up, the tension might be about enough to balance the weight, whereas, on the spans at the upper end the tension is often enough to produce an uplifting effect on the carrier. On systems

---

[8] Derivation of short-cut formula used

$$\text{Vertical component} = \text{Total weight x sin. } \alpha$$
$$= \text{Weight per foot x span x } \frac{\text{Diff. elev.}}{\text{span}}$$
$$= \text{Weight per foot x diff. elev.}$$



For overhead rollers – $H = \frac{Wmn}{2(t_1 \quad t_2) \cos a}$

$t_1$ = tension in track cable

$t_2$ = tension in traction cable

$m = n = 1/2\ S$

$a = 0$

Figure 9.– Relative position of traction cable for overhead and low rollers on a horizontal span.

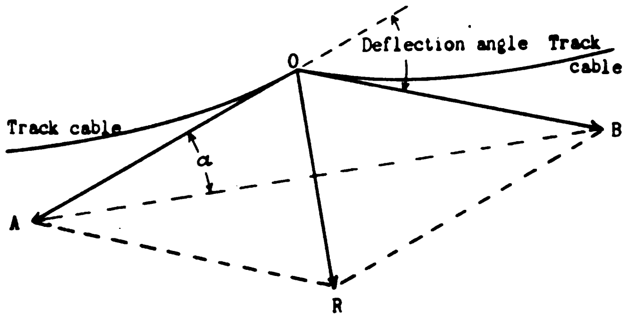


Figure 10.– Force diagram showing forces acting at O, the point of support of the track cable. OA and OB are tangent to the cable at the point of support and are equal to the tension in the cable.

In the parallelogram OARB,

$OA = OB = \text{tension in cable} = T$

$OR = \text{resultant} = 2\ T \sin a$

where $a$ = one-half the deflection angle.

where there is a great difference in elevation between the highest and lowest points, it is very difficult to avoid some uplifting effect of the traction cable at the upper end. It might be enough that there would be a tendency to lift the carriers off the track cables, but usually it is not that serious. A good empirical rule to follow is to design the system so that the sag of the empty track cable would never be more than the sag of the empty traction cable on the same span. Deflections for empty track and traction cables on any span can be calculated by finding the tension in each by formula (2) and then substituting these values in formula (1) to find the maximum or center deflection.

With overhead rollers(see fig. 9), the cables are practically the same distance apart at the supports as at the loads, and the deflection can be calculated as if the two cables had common points of support and loading, as follows:

In formula $H = \dfrac{wmn}{2t\cos\alpha}$ (1)

H = deflection

w = distributed weight of loads per foot + weight of track cable per foot + weight of traction cable per foot

m = distance from left support

n = distance from right support

t = tension in track cable + tension in traction cable

$\alpha$ = vertical angle of the inclined chord.

The tension in the traction cable for any span is found by formula (2).

Where the rollers on the towers are placed low, so that the carriers pass over them, a greater proportion of the weight of the traction cable falls on the track cable than is the case for overhead rollers. In such cases, the effect of the traction cable on the sag becomes more complicated and is the greatest at the carrier nearest the support. For flat profiles, where the tension in the traction cable is low or moderate, this effect is very slight and deflections may be calculated by considering the weight of the traction cable a part of the loading on the track cable at the carriers and calculating the deflections accordingly. On the crests of ridges, where a number of towers must be placed at close intervals or where curved rail structures are necessary, overhead rollers should always be used.

In all of the following calculations in this paper, the weight of the traction cable is neglected.

## Calculations of Cable Deflections

The size of the track cable required for the loaded side has been determined as 1-3/8 inches in diameter. For carriers weighing 570 pounds and having a capacity of 1,200 pounds and spaced at 192 feet, the distributed loading per foot on the loaded side is as follows:

Loaded carrier - 1,770 ÷ 192 = 9.22 pounds per foot.

1 3/8 inches locked cable weighs 4.56 pounds per foot.

Total weight is 13.78 pounds per foot.

For the span from the discharge terminal to the first tower the horizontal distance is 1,400 feet.

In the formula $H = \dfrac{wmn}{2t \cos \alpha}$ (1)

H = unknown = deflection at any point.

W = 13.78 = weight per foot of cable plus the distributed load.

M = 200 = distance from point to left support.

N = 1,200 = distance from point to right support.

t = 47,000 = working tension of track cable (furnished by manufacturer).

a = 17°04' = vertical angle from support at loading terminal to support at structure at horizontal coordinate 1400.

$H = \dfrac{13.78 \times 200 \times 1{,}200}{2 \times 47{,}000 \times \cos 17°04'} = 36.8$ = deflection at horizontal coordinate 200.

From this formula, deflections at each 200-foot interval are found to be as follows:

| | | | |
|---|---|---|---|
| At horizontal coordinate | 200, | - 36.8 | feet from the inclined chord. |
| Do. | 400, | - 61.3 | do. |
| Do. | 600, | - 73.6 | do. |
| Do. | 700, | - 75.1 | do.(center deflection) |
| Do. | 800, | - 73.6 | do. |
| Do. | 1,000, | - 61.3 | do. |
| Do. | 1,200 | - 36.8 | do. |

Since the difference in elevation of the points of support of the span from the discharge terminal to the first tower is 430 feet, the difference in elevation of points on the inclined chord at 200-foot intervals is 430 ÷ 7, or 61.43 feet. The elevation of the cable at each interval is as follows:

Elevation of support at discharge terminal = 9,620.

At horizontal coordinate 200, 9,620 + 61.4 - 36.8 = 9,644.6
Do. 400, 9,620 + 122.9 - 61.3 = 9,681.6
Do. 600, 9,620 + 184.3 - 73.6 = 9,730.7
Do. 800, 9,620 + 245.7 - 73.6 = 9,792.1
Do. 1,000, 9,620 + 307.2 - 61.3 = 9,865.9
Do. 1,200, 9,620 + 368.6 - 36.8 = 9,951.8

## Corrections for Variation in Tension and Concentrated Loading

These deflections and the corresponding elevations are sufficiently accurate for all practical purposes in tramway design. They are not entirely correct because of certain assumptions regarding the formula and the manner in which it is applied. The differences in the deflections and elevations as shown, and the true values for these quantities, are very small and are well within the allowable limits required for tramway design. The assumptions to which these differences are due are, first, the formula $H = \frac{wmn}{2t \cos \alpha}$ is the formula of a parabola, whereas the theoretical curve formed by a cable suspended between two supports is a catenary. The formula of a parabola assumes uniform loading along the inclined chord, whereas the loading is actually uniform along the curve. Second, it is assumed that the tension is the same along all parts of the span, whereas it actually increases from the lower support to the upper support, due to the cable being suspended in an inclined position. Third, it is assumed that the loading is uniformly distributed along the whole span, whereas the loading due to the carriers is actually concentrated at points 192 feet apart on the curve.

The first of these assumptions can be disregarded entirely because deflections calculated by the parabolic formula and the catenary formula will check to within an inch, except for extremely large deflections that ordinarily would not occur in tramway construction.

Differences in deflections due to difference in tension in the track cable along the inclined span are small and unimportant, but if the true deflections are desired they can be found by correcting for tension at each point and recalculating by formula (1) $H = \frac{wmn}{2t \cos \alpha}$.

On the span from the discharge terminal to the first tower, the tension at the lower support on the loaded side is the applied tension of 47,000 pounds. The tension at any point on this span is the tension at the lower support plus the weight of the cable per foot multiplied by the difference in elevation of that point and the lower support.

Thus, elevation at lower support = 9,620.0

elevation at horizontal coordinate 200 = 9,644.6

Difference = 24.6

Tension at horizontal coordinate 200 = 47,000 + 4.56 x 24.6 = 47,112.

In this way the actual tension at each 200-foot interval is found to be as follows:

| | | | |
|---|---|---|---|
| At horizontal coordinate | 200, | 47,112 | pounds |
| Do. | 400, | 47,281 | do. |
| Do. | 600, | 47,505 | do. |
| Do. | 800, | 47,785 | do. |
| Do. | 1,000, | 48,120 | do. |
| Do. | 1,200, | 48,513 | do. |

By using the actual or corrected tension in formula (1), $H = \frac{wmn}{2t \cos \alpha}$, the deflections at each point can be recalculated. The results of such recalculations are shown herewith, and for comparison the results of the calculations in which it is assumed the tension is constant are entered in the right-hand column.

| Horizontal coordinate | Deflections calculated for variable tension | Deflections calculated for uniform tension |
|---|---|---|
| 200 | 36.7 | 36.8 |
| 400 | 60.9 | 61.3 |
| 600 | 72.8 | 73.6 |
| 800 | 72.3 | 73.6 |
| 1,000 | 59.9 | 61.3 |
| 1,200 | 35.6 | 36.8 |

The curve for the tension corrected at each point is slightly flatter at the upper end than the curve determined by assuming uniform tension. The maximum difference in this case is only 1.4 feet, which is well within the allowable limit of error for this feature in tramway design. On extremely long spans with steep inclination this difference may be as much as 4 or 5 feet, in which case it might be important in determining the location and height of intermediate supports.

For spans on which there are three or more concentrated loads, errors due to the assumption of uniformly distributed loading are of even less amount than errors due to the assumption of uniform tension. Formulas for deflections at points of concentrated loading have been worked out by Fred C. Carstarphen of Denver.[9] These formulas involve the use of the principal of second differences, which is very ably discussed by Mr. Carstarphen in the publication referred to. The principal of second difference as applied to cable formulas and cable deflections is, briefly, this: The differences in the deflections at points of successive concentrated loading are variable, but the difference of these differences, or the second differences, is constant in each case.

In applying the principal of second differences, the center deflection is calculated for the loads distributed symmetrically with respect to the center of the span. The second differences and the first difference at the center loads are calculated and the rest of the deflections are found by addition and subtraction. Three formulas for deflections and one for second differences are shown here.

Case 1. Odd number of loads with one load at center of span.

Deflection at the center is

$$y = \frac{gs}{4t}\left[n - \frac{n^2-1}{2}q\right] + \frac{ws^2}{8t} \qquad (3)$$

Case 2. Even number of loads, with one of the center loads placed $\frac{a}{4}$ beyond the center of span. The deflection at that load is

$$y = \frac{gs}{4t}\left[n - \frac{n^2}{2}q + \frac{n}{4}q^2\right] + \frac{ws^2}{2t}\left[1/4 - \frac{1}{16}q^2\right] \qquad (4)$$

Case 3. Even number of loads placed symmetrically to the center of the span, deflection at center is

$$y = \frac{gs}{4t}\left[n - \frac{n^2}{2}q\right] + \frac{ws^2}{8t} \qquad (5)$$

---

[9] Carstarphen, F. C., A Simple Method of Computing Deflections of a Cable Span Carrying Multiple Loads Evenly Spaced: Colorado Sch. Mines Mag., vol. 19, no. 12, December 1929, p. 15.

The second difference is

$$\frac{a}{t}\left[g + wa\right] \qquad (6)$$

In these formulas,

$g$ = weight of concentrated load.

$y$ = deflection from inclined chord.

$n$ = number of loads.

$s$ = horizontal distance between supports.

$t$ = tension.

$a$ = horizontal distance between loads.

$q = \frac{a}{s}$.

$w$ = weight per foot of cable divided by the cosine of the vertical angle between supports.

Note that the formula for second difference is not actually a constant, except for horizontal spans as it involves the tension t, which, as we have already seen, is variable for inclined spans. In the following calculations it is assumed to be constant.

Ordinarily only the first and the last of these formulas need be used. The other two might be useful for solving special problems.

Applying equation 2 to the span from horizontal coordinate 0 to coordinate 1400 (fig. 8), we have:

$g$ = weight of loaded carrier = 1,770 pounds.

$n = 7$.

$s = 1,400$.

$t = 47,000$.

$a$ = horizontal spacing of loads

$= 192 \times \cos 17°04' = 184$ feet

$$w = \frac{4.56}{\cos 17°04'} = 4.77.$$

$$q = \frac{184}{1400} = .1314$$

With one load in the center, there will be seven loads on the span spaced at 148, 332, 516, 700, 884, 1,068 and 1,252 feet from the left support.

Substituting in formula (3), we have

$$y = \frac{1,770 \times 1,400}{4 \times 47,000} \left[ 7 - \left( \frac{49-1}{2} \right) .1,314 \right] + \frac{4.77 \times 1,400 \times 1,400}{8 \times 47,000}$$

$$= 50.7 + 24.8$$

$$= 75.5 = \text{center deflection from inclined chord,}$$

and from formula (5),

$$\text{second difference} = \frac{184}{47,000} \quad (1,770 + 4.77 \times 184)$$

$$= 10.37$$

The first difference at the center of the span is one-half the second difference, or 5.18.

In the table shown herewith, there are seven columns with the center deflection entered in the fourth or center column of the fourth row. The second differences are entered in all of the columns of the second row and the first difference for the center loads, between the third and fourth and the fourth and fifth columns of the third row. In filling out the table, it is necessary to start from the center and work both ways. The rest of the first differences are calculated by adding the second difference to each preceding first difference, and the rest of the deflections are calculated by subtracting each second difference from the preceding deflection. For comparison, deflections calculated by assuming uniform loading and uniform tension are entered in the fifth row, and deflections calculated by assuming uniform loading but corrected for tension at each load, are entered in the sixth row.

| Distance from left support.. | 148 | 332 | 516 | 700 | 884 | 1,068 | 1,252 |
|---|---|---|---|---|---|---|---|
| Second difference........... | 10.37 | 10.37 | 10.37 | 10.37 | 10.37 | 10.37 | 10.37 |
| First difference............ | 25.92 | 15.55 | 5.18 | 5.18 | 15.55 | 25.92 | |
| Deflections calculated by method of second difference. Tension = 47,000............ | 28.9 | 54.8 | 70.3 | 75.5 | 70.3 | 54.8 | 28.9 |
| Deflections calculated by assuming uniform loading. and uniform tension. Tension = 47,000............ | 28.4 | 54.4 | 70.0 | 75.1 | 70.0 | 54.4 | 28.4 |
| Deflections calculated by assuming uniform loading but corrected for variation in tension. Tension varies from 47,000 to 48,623...... | 28.4 | 54.2 | 69.4 | 74.1 | 68.6 | 53.0 | 27.5 |

## Comparison of Methods of Calculations

The two methods for calculating deflections and the method for correcting for variations in tension in an inclined span are given here to show the close approximation of all results. The formulas used in connection with the principal of second difference, although independently derived, are fundamentally the same as formula (1) $H = \frac{wmn}{2t \cos \alpha}$. The apparent differences are only refinements taking into account the effect of concentrated loading. From the foregoing table, it is apparent that differences in deflection due to variations in tension on a steeply inclined span are more important than differences due to concentrated loading.

The effect of concentrated loading becomes more important where carriers of large capacity are used. In such cases, it would be advisable to use formulas 2, 3, or 4, especially for spans where it is necessary to determine the deflection with more than ordinary accuracy.

All subsequent calculations in this paper will be based upon the assumption of uniform tension and uniform loading.

## Determination of Height and Location of Towers

The cable elevations for the whole line are calculated and plotted on the profile (fig. 8). From this profile, it is apparent that there is sufficient clearance at all points except at horizontal coordinate 2200. The ground elevation at this point is 10,210 feet and the cable elevation as calculated is 10,222.1 feet. This leaves a clearance of only 12.1 feet, which is not sufficient. A tower is located at this point and the cable elevations are recalculated and plotted accordingly. On the profile, the position of the cable without the tower is shown by a broken line and the position with the tower in place by a full line.

The proper location for intermediate structures is not always as apparent as this, and sometimes a more detailed study of the profile is required to determine the most advantageous location.

Ordinarily, a tower must be only high enough to provide a minimum ground clearance. However, the general ground profile from horizontal coordinate 1400 to horizontal coordinate 2800 is concave. This complicates the problem, owing to the fact that if the tower is of minimum height necessary to provide ground clearance only, the cables will lift out of the saddle when the line is partly unloaded or during strong winds.

There is no definite rule for determining the height of such towers. Some tramway engineers 10/ plot a construction curve determined by using a tension equal to 1 1/2 times the working tension and build the towers up to that curve.

---

10/ Durham, Edward B., Aerial Tramways and Cableways: Mining Engineers' Handbook by Peele, 2nd ed., 1927, vol. 2, sec. 26, p. 1745.

The elevation of the construction curve at horizontal coordinate 2200 is determined as follows:

Working tension = 47,000 pounds

Tension for construction curve = 1 1/2 times the working tension = 70,500 pounds.

Inclination of the chord from support at horizontal coordinate 1400 to horizontal station 2800 = $\tan^{-1} \frac{10,480 - 10,050}{2,800 - 1,400} = 17^\circ 04'$.

Using formula (1), deflection at horizontal coordinate 2200 = H =

$$\frac{13.78 \times 800 \times 600}{2 \times 70,500 \times \cos 17^\circ 04'} = 49.1 \text{ feet.}$$

Elevation of construction curve
at horizontal coordinate 2200 = $10,050 + 800 \tan 17^\circ 04' - 49.1$
= 10,246.6

Ground elevation at horizontal
coordinate 2200 = 10,210.0
Difference = 36.6, which is the height of tower necessary to build up to the construction curve.

Calculation of Weight on Towers

The weight of cables on a tower can be calculated by first calculating the angle of deflection at the tower and then finding the resultant force due to the tension in the cables. The deflection angle is calculated by the formula $\tan \phi = \frac{4H \pm V}{S}$ (7) [11] in which $\phi$ is the vertical angle of the tangent to the cable at the point of support. V is the vertical distance between the ends of the span, S is the span, and H is the deflection at the center.

For the tower at horizontal coordinate 2200, consider first the vertical angle of the tangent to the cable on the upper side of the tower.

S = 600

V = 10,480 - 10,246.6 = 233.4.

H = the cable deflection at the center of the span and is calculated from formula (1) (using a working tension of 47,000 pounds) and is found to be 14.2 feet. (Tower changes $\alpha$ from 17°04' to 21°16'.)

---

[11] Peele, Robert, Formula (7): Mining Engineers' Handbook, 2d ed., 1927, p. 1747.

Then, $\tan \phi = \frac{4 \times 14.2 - 233.4}{600} = \frac{-176.6}{600}$

and $\phi = 16°24'$ (the negative sign of the tan. shows that the angle is the negative of a negative angle, or in other words a positive angle.)

For the angle on the lower side of the tower:

$S = 800$

$V = 10,246.6$ feet - $10,050$ feet = $196.6$ feet

H calculated from formula (1) = 24.2 feet (tower changes $\alpha$ from 17°04' to 13°48').

Then $\tan \phi_2 = \frac{4 \times 24.2 + 196.6}{800} = \frac{293.4}{800}$

and $\phi_2 = 20°08'$.

Therefore, the deflection angle is 20°08' - 16°24', or 3°44', and the angle between tangents is 180°00' - 3°44' = 176°16'.

In figure 10, OA is the tangent to the cable on the lower side of the tower and OB the tangent to the cable on the upper side of the tower. If OA and OB are equal, and each represents the tension in the cable, then OR of the parallelogram OARB will represent the direction and magnitude of the resultant R, which is also the weight on the tower due to the cable. This is solved algebraically by the formula $R = 2T \sin a$, (7a), in which 'a' equals one-half the deflection angle.

For the tower at horizontal coordinate 2200, $a = \frac{3°44'}{2} = 1°52'$

$T = 47,000$ pounds

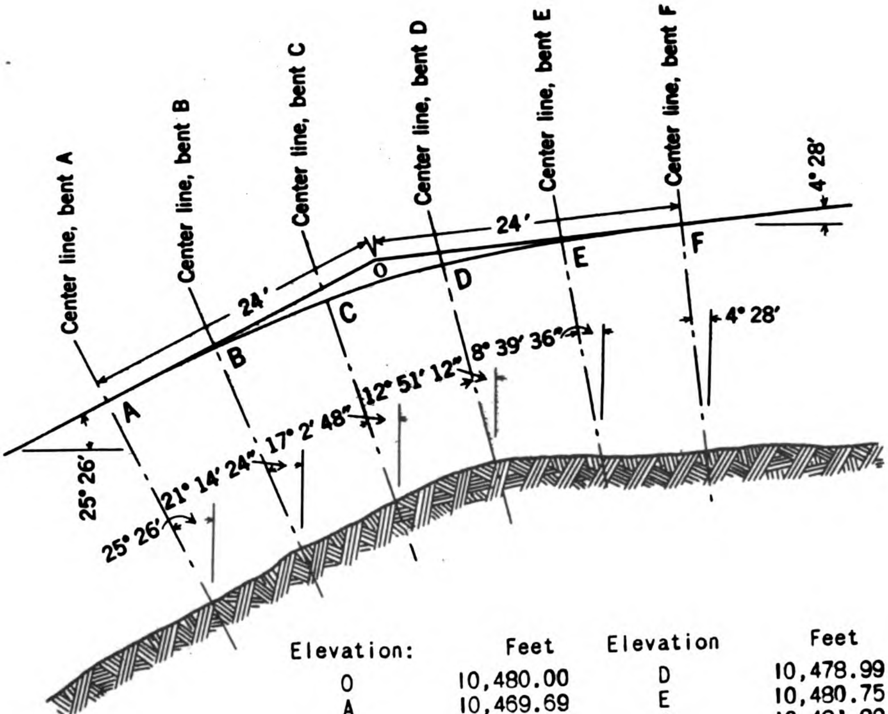
Then $R = 3,055$ pounds.

This is the weight on the loaded side of the tower, assuming uniformly distributed weight on the cable. It varies as the loads approach and leave the tower.

## Design of Rail Structures

From formula (7) it is found that the deflection angles of structures at horizontal stations 1400 and 2800 are 20°22' and 21°31', respectively, and, hence, rail structures or break-over towers must be used at those points instead of towers having one point of support only.

In designing a break-over tower, it is first necessary to decide upon the degree of curvature and the length. Each of these factors will depend upon the other and both will depend upon the nature of the ground where it



| Elevation: | Feet | Elevation | Feet |
|---|---|---|---|
| O | 10,480.00 | D | 10,478.99 |
| A | 10,469.69 | E | 10,480.75 |
| B | 10,473.45 | F | 10,481.82 |
| C | 10,476.58 | | |

Chords AB, BC, CD, DE, and EF = 9.48 feet

Figure 11.—Typical rail structure location.

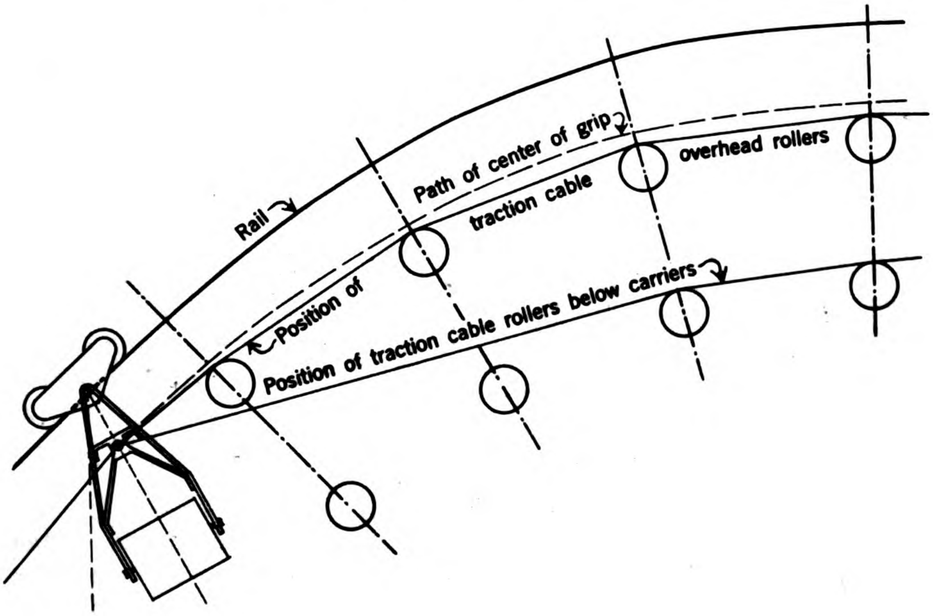


Figure 12.—Sketch showing relative position of traction cable and carrier for overhead rollers and low rollers on rail structure of short radius.

is to be built. If the crest of the ridge is very sharp, it may be necessary to use a short structure with a short radius of curvature; whereas, if the crest is long and rounded, it would be more desirable and probably necessary to use a longer one with a longer radius of curvature.

Break-over towers with large radii of curvatures are more desirable, but their cost is greater. Therefore, it would seem advisable to use as large a radius of curvature as is consistent with construction costs of the tram as a whole.

Once the radius of curvature is decided upon, the length of the structure and the elevations of the saddles are calculated very much the same as the length and coordinates of a circular railroad or highway curve. In some cases, it may be desirable to decide on the length first and from this calculate the radius of curvature.

In figure 11, OA and OF are the tangents to the cable at the assumed point of support at horizontal coordinate 2800. Since the deflection angle is $21^{\circ}31'$ and we wish to build a break-over tower having points of support at each of which the cables will be deflected from $3\ 1/2^{\circ}$ to $4\ 1/2^{\circ}$, a tower of six bents or six points of support will be required. If these points of support are placed about 8 feet apart, the structure will be about 48 feet long, measured along the curve of the rail, or, to decide upon something definite, say 24 feet on each side of the assumed support at O, measured along the tangents. It must be designed so that the cables will be, as nearly as possible, tangent to the curve at points A and F. The vertical angles of the tangents to the cables at these points will be slightly different from those of the tangents at the point O, but for all practical purposes, we could use the latter. However, the true tangents can be very closely approximated from the formula

$$\tan \phi = \frac{2d \sin \alpha \pm H}{2d \cos \alpha} \qquad \text{(8) approximate,}$$

in which $\phi$ is the vertical angle of the tangent to the cable at a **given distance** from the point of assumed support—in this case 24 feet. $\alpha$ is the vertical angle of the inclined chord from the assumed support to the support at the other end of the span—$21^{\circ}15'$ on the lower or left side of the support and $+8^{\circ}40'$ on the upper or right side of the support. H is the deflection of the cable at a distance 2d or 48 feet from the point of support, calculated by formula (1). $H = \frac{wmn}{2t \cos \alpha}$.

Since 2d is measured along the tangents to the curve, it must be reduced to a horizontal distance for computing the deflection H. Since the angles of the tangent are not known, the angles of the inclined chord will be sufficiently close.

$$48 \times \cos 21^{\circ}15' = 44.7$$

$$48 \cos 8^\circ\text{-}40' = 47.5$$

$$H = \frac{13.78 \times 44.7 \times 555.3}{2 \times 47{,}000 \times \cos 21^\circ 15'} = 3.9 \text{ at 48 feet to left of assumed support.}$$

$$H = \frac{13.78 \times 47.5 \times 482.5}{2 \times 47{,}000 \times \cos 21^\circ 15'} = 3.4 \text{ at 48 feet to right of assumed support.}$$

For the vertical angle of the tangent to the cable at 24 feet to the left of O.

$$\tan \phi = \frac{2 \times 24 \sin 21^\circ 15' + 3.9}{2 \times 24 \cos 21^\circ 15'} \qquad (8)$$

and $\phi = 25^\circ 26'$.

For the vertical angle of the tangent to the cable at 24 feet to the right of O.

$$\tan \phi = \frac{2 \times 24 \sin 8^\circ 30' - 3.4}{2 \times 24 \cos 8^\circ 30'} \qquad (8)$$

and $\phi = 4^\circ 28'$.

The total deflection angle = $25^\circ 26' - 4^\circ 28' = 20^\circ 58'$, and the deflection angle at each bent or point of support = $\frac{20^\circ 58'}{5} = 4^\circ 11'\ 36''$.

$$\text{The radius of curvature} = \frac{24}{\tan 1/2 \text{ the total deflection angle}}$$

$$= \frac{24}{\tan 10^\circ 29'} = 129.7 \text{ feet.}$$

The distance between the centers of the bents at their points of intersection with the curve is the radius of curvature x 2 sin 1/2 the central angle for each bent or 129.5 x 2 sin $2^\circ 05'\ 48''$ = 9.48 feet.

If the structure is built so that each bent will be normal to the tangent of the curve at its point of intersection, the first bent (bent A on the sketch, fig. 11) will be inclined at an angle of $25^\circ 26'$ from the vertical, as shown. The inclination of the rest of the bents will be as follows:

Bent A...........$25^\circ 26'$

Bent B...........$25^\circ 26' - 4^\circ 11' 36'' = 21^\circ 14' 24''$

Bent C...........$21^\circ 14' 24'' - 4^\circ\ 11' 36'' = 17^\circ 02' 48''$

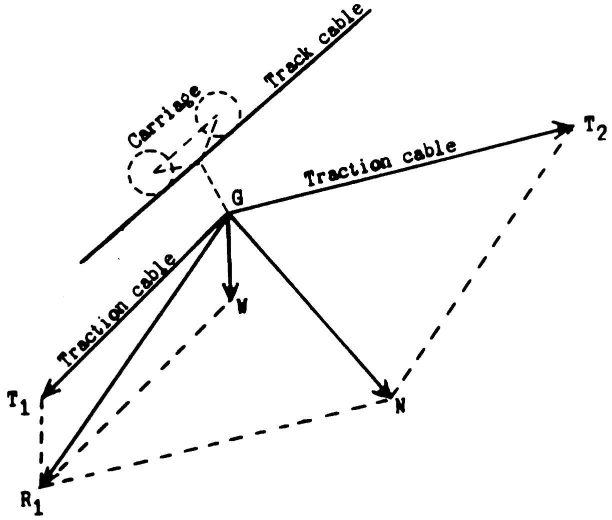


Figure 13.— Force diagram of figure 12 for low rollers.

GW = weight of loaded carrier,

$GT_1$ = tension in traction cable below grip G,

$GR_1$ = resultant of $GT_1$ and GW,

GN is normal to the track cable and $GT_2$ is the position of traction cable.

In the parallelogram $GR_1NT_2$,

GN = force normal to track cable and represents the strain on the carriage,

$GT_2$ = tension above the grip G.

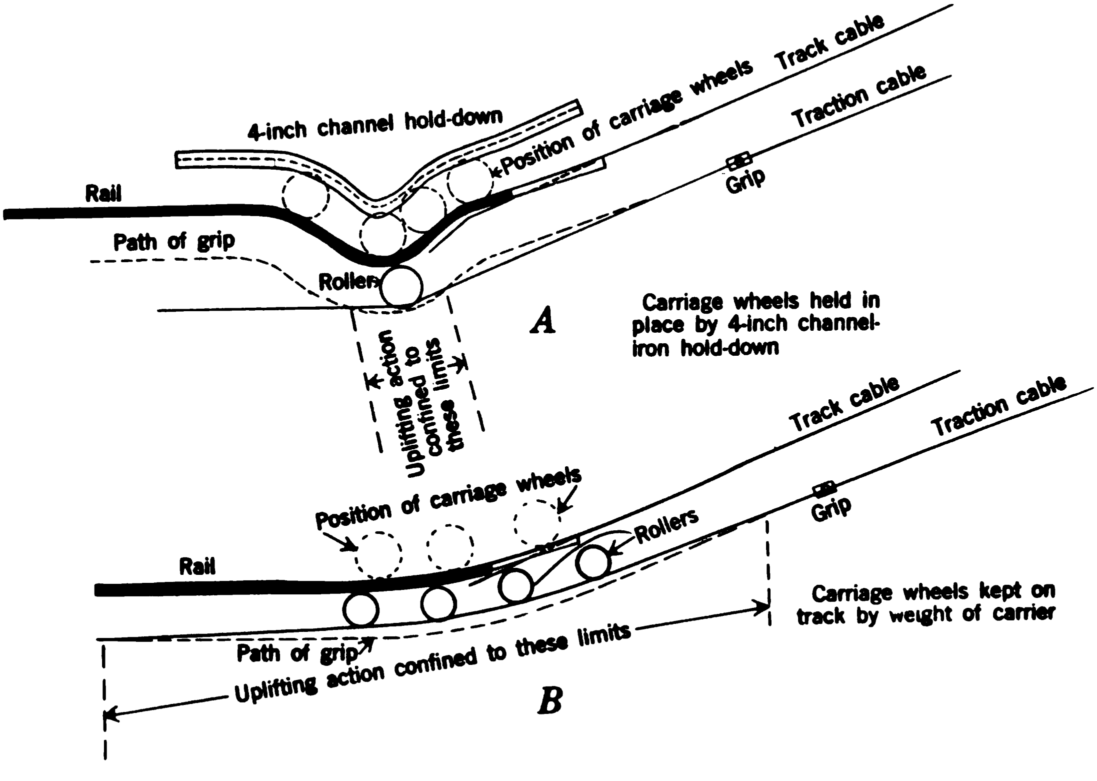

4-inch channel hold-down
Position of carriage wheels
Track cable
Traction cable
Rail
Grip
Path of grip
Roller
A
Carriage wheels held in place by 4-inch channel-iron hold-down
Uplifting action confined to these limits
Track cable
Traction cable
Position of carriage wheels
Grip
Rail
Rollers
Carriage wheels kept on track by weight of carrier
Path of grip
Uplifting action confined to these limits
B

Figure 14.— Hold-down rollers used at terminals at the lower end of steeply inclined spans.

Bent D...........17°02'48" - 4°11'36" = 12°51'12"

Bent E...........12°51'12" - 4°11'36" = 8°39'36"

Bent F...........8°39'36" - 4°11'36" = 4°28'00"

Note that the inclination for bent F is also the vertical angle of the tangent to the cables at 24 feet to the right of the assumed support. The elevations of points on the curve at the center of each bent or each point of support are calculated from the elevation of the assumed point of support at O, and from the deflection angles at each of these points.

Elevation of O (see fig. 8) = 10,480.00
and, from figure 11,

Elevation A = 10,480.00 - 24 sin 25°26' = 10,469.69

Elevation B = 10,469.69 + 9.48 sin (25°26' 00" - 2°05'48") = 10,473.45

Elevation C = 10,473.45 + 9.48 sin (21°14'24" - 2°05'48") = 10,476.56

Elevation D = 10,476.56 + 9.48 sin (17°02'48" - 2°05'48") = 10,479.01

Elevation E = 10,479.01 + 9.48 sin (12°51'12" - 2°05'48") = 10,480.78

Elevation F = 10,481.87 + 9.48 sin (8°39'36" - 2°05'48") = 10,481.87

Also, the elevation of F = 10,480.00 + 24 sin 4°28' = 10,481.87.

From these elevations and from the topography at the location, the structure is designed and foundation plans drawn up. The posts may be vertical or inclined so that they will be perpendicular to the tangents of the curve at the points of intersection with the center lines. If the cables enter either end at an angle of 25° or more, the latter arrangement is more satisfactory and more economical because less bracing is required.

## High and Low Rollers

On most break-over towers, the rollers for the traction cable are placed high enough so that they will be just under the grips as the carriers pass. These are commonly called overhead rollers, as distinguished from rollers that are placed at some distance below the carriers. The object of overhead rollers is to eliminate excessive stresses in the cables and carriages. Figure 12 shows a carrier approaching a rail structure of short radius, and figure 13 shows a graphic solution of the forces acting on the cables when the rollers are below the carriers. From this, it is apparent that the force acting on the cable and the carriages due to tension in the traction cable is nearly twice as great as the force due to the weight of the load alone and the total force is three times as great as the weight of the load. Also, the tension in the traction cable is about 20 percent greater than the normal tension would be. Figure 12 also shows the position of the traction cable for overhead and low rollers. When overhead rollers are used, the traction cable is kept nearly parallel to the track cable for all positions of the carrier as it approaches and, hence, practically all excessive strains are eliminated.

## Special Problems

A rather difficult problem arises when the cables enter a terminal at a steep negative angle. If there is much tension in the traction cable, there is a tendency to lift the carriers off the cable or the rail as they enter or leave the terminal. Figure 14 shows two ways in which this uplifting action is taken care of. At A, the traction cable is kept parallel to the track cable by a roller placed very low, and the dip in the rail brings the carrier down so that the grip passes under the roller. By this arrangement there is no uplift except right at the dip, and here the carrier is held in place by a channel iron hold-down above the carriage wheels. This method is efficient but objectionable because it causes the carriers to swing annoyingly as they enter and leave the terminal.

The arrangement shown at B has been found to be satisfactory when the tension in the traction cable is not too great, although there is an uplifting action over a considerable distance and in places where it is impractical to place a guide or a hold-down. It works successfully without guides or a hold-down of any kind in an installation in Colorado where the cables enter the lower terminal at an angle of about 28°; but in this case the terminal is at the lower end of the system, where the traction tension is at a minimum. With proper guides and hold-downs where the uplifting action is most intense, it would undoubtedly work with a much greater traction tension.

## POWER

The power developed or required by a tram is calculated by the formula

$$HP = \left(\frac{WV}{33,000} \pm f\,L.\right) S \qquad (9).$$

Where W = the distributed weight of the loads only.

V = the vertical distance between terminals.

S = the speed of the traction cable in feet per minute.

f = coefficient of friction.

L = the total weight of all moving parts (loaded carriers, empty carriers, traction cable, and terminal machinery).

The term fL, is negative when power is required and positive, when power is generated.

Friction may be divided into line friction and terminal friction. The line friction is due to the rolling friction of the carriage wheels and the bearing friction in the carriage wheels and the rollers on the structures. The coefficient of friction for each of these items and also for terminal machinery is necessarily different. The coefficient of friction, as used in the formula, is, therefore, the coefficient for the system as a whole. Power required to overcome friction will vary from about 1/2 percent to 2 1/2 percent of the total power required.

## CONDITIONS AFFECTING CONSTRUCTION COSTS

The cost of aerial tramways varies greatly for different sections of the country. Tramways having capacities of 35 to 50 tons per hour have been built for as little as $4.25 per foot.[12] This cost can be approached only in countries where it is possible to get an abundance of skilled labor at a reasonable rate, and where it is possible to deliver all materials at the points where they are to be used by truck or some other low-cost method. The other extreme is probably represented in southwestern Colorado, where there is a scarcity of almost everything required and where much of the material for construction purposes (even water for concrete) must be delivered by pack animals. In a country such as this, a dependable tram will cost $10 per foot or more. A lighter and less substantial tram might be built for $8 a foot, but such a tram could not be depended on to operate successfully during the severe storms that prevail in that country during the winter months. Other things being the same, a short tram will cost more per foot than a long tram, due to the fact that the terminals represent such a large percentage of the total cost.

The following table [13] shows the cost of equipment for tramways of a standard American make for capacities of 25 to 100 tons per hour. The data do not include the cost of structures or installation costs. For lines requiring braking and control equipment, $175.00 to $850.00 must be added.

---

[12] This cost based on new material.

[13] Carstarphen, F. C., Aerial Tramways: Trans. Am. Soc. Civil Eng., vol. 92, pp. 875-973.

| Capacity per hour | Size cable loaded side, inches | Size cable empty side, inches | Size traction rope, inches | Couplings per mile | Capacity carriers, cubic feet | Spacing of carriers | Time interval, seconds | Number of carriers per mile | Number of supports | Weight, in pounds per 100 feet | Cost per 100 feet | Terminal weight, pounds | Terminal cost | Tension station weight, pounds | Tension station cost |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 1 | 7/8 | 1/2 | 4 | 5 | 300 | 36 | 37 | 20 | 1,017 | $144 | 26,400 | $2,894 | 7,300 | $733 |
| 30 | 1 1/8 | 7/8 | 1/2 | 4 | 6 | 300 | 36 | 37 | 22 | 1,127 | 155 | 32,700 | 3,540 | 7,800 | 766 |
| 40 | 1 1/8 | 7/8 | 1/2 | 4 | 6 | 225 | 27 | 48 | 22 | 1,191 | 172 | 32,700 | 3,540 | 7,800 | 766 |
| 50 | 1 1/8 | 7/8 | 1/2 | 4 | 6 | 180 | 21.6 | 59 | 22 | 1,256 | 189 | 32,700 | 3,540 | 7,800 | 766 |
| 50 | 1 1/4 | 7/8 | 5/8 | 5 | 12 | 360 | 43.2 | 32 | 22 | 1,348 | 184 | 38,900 | 4,154 | 9,100 | 900 |
| 100 | 1 1/4 | 7/8 | 5/8 | 5 | 12 | 180 | 21.6 | 60 | 22 | 1,625 | 243 | 38,900 | 4,154 | 9,100 | 900 |
| 100 | 1 3/8 | 7/8 | 5/8 | 6 | 15 | 225 | 27 | 49 | 22 | 1,703 | 240 | 38,900 | 6,087 | 10,100 | 1005 |

Construction and installation costs vary greatly according to topography and the availability of labor and material. They may amount to as little as the equipment costs in countries where the topography is favorable and the supply of labor and material is plentiful. In regions less favorable they may amount to two or three times the equipment costs.

## CONCLUSION

Nearly all of the large manufacturers of aerial tramways maintain efficient engineering staffs and can furnish accurate cost data to the prospective buyer. It should be remembered, however, that these data are based largely upon the costs of materials furnished by the manufacturer and his experience in building tramways. If the estimate is for a tram in a region with which the manufacturer is not familiar, it is possible that this estimate will not be accurate. A very good example of this is the case of a manufacturer making an estimate of $60,000 for a job that ultimately cost $98,000.

The manufacturers of aerial tramways make every effort to provide a customer with the best installation for his needs. However, unless a prospective buyer is familiar with the economic and engineering features of tramway construction (which usually he is not), it would appear prudent for him to engage a competent tramway engineer to study his problem and advise him in making the purchase.